UNSTEADY FLOW OF DUSTY FLUID THROUGH CIRCULAR PIPE: A STUDY

Smt. N.G. Siddagangamma

Assistant Professor, Department of Mathematics, Siddaganga Institute of Technology, Tumkur, Karnataka-572013, India.

ABSTRACT

The present paper an attempt to study flow of an unsteady dusty fluid through a circular pipe under the influence of impulsive pressure gradient. We have assumed that the initial fluid velocity to be some average velocity. Analytical expressions for the velocities of fluid and dust particles are obtained by using Laplace transform technique. The influence of mass concentration and relaxation time of the dust on the velocity have been clearly pointed out. Results have been discussed with the help of graphs.

KEYWORDS: Pipe flow; Two-phase flow; Non-Newtonian fluid; incompressible flow; volume fraction.

1. INTRODUCTION:

The mechanical behaviour of dusty fluids has been a subject of study receiving greater attention during the recent past of several researchers in the field of Fluid Dynamics. Problems dealing with the influence of dust particles on viscous flows find place in several branches of Science and Technology. Some such flows are those of red corpuscles and other bodies in blood, fluidization, combustion, environmental pollution and chemical engineering flow in rocket tubes where small carbon or metallic fuel particles are present and so on.

Since a dusty fluid is a mixture of fluid and fine dust particles, its study is of practical importance. Particularly, the study of flow of dusty fluids through circular pipes is of great importance which has applications in various Industries. Considering blood as a two-phase fluid, this study also gives some insight into the blood flow through veins or arteries

Interest in problems of mechanics of systems with more than one phase has developed rapidly in recent years. Much work has already been done on dusty fluid flow. The equation n of motion of fluid carrying small particles were first derived. P.G.Saffman (1962) who investigated the stability of the laminar flow of a suspension in which the particles are uniformly distributed. A.H. Nayfeh (1966) introduced equation of motion of fluid containing dust particles taking volume fraction of dust particles into account. S.Uchida (1956) has made some theoretical analysis of pulsating viscous flow in circular pipe. S.K.Nag et al., (1981) have considered the flow through an elastic tube rather than a rigid tube. Kishore and Pandey (1977) have discussed the same problem but they found the effect of particles size on velocity of sedimentation. Datta and Dalal (1992) have discussed the unsteady flow of dusty fluid through a circular pipe. S.N. Dube and Srivastava (1972) have discussed the unsteady flow of dusty viscous fluid in a channel and pipe under the influence of linear pressure gradient. Sambasiva Rao (1969) has obtained the analytical solutions for the dusty fluid flow through a circular tube under the influence of constant pressure gradient. S. Rashmi et.al. (2007) have discussed the unsteady flow of a dusty fluid between two oscillating plates under varying constant pressure gradient and Siddapasappa et.al. (2008) have discussed the viscous dusty fluid flow with constant velocity magnitude.

In all the above discussed papers the authors considered the zero initial velocity, but in this paper, the authors study the problem of unsteady flow of dusty fluid through a circular pipe induced by an impulsive pressure gradient taking into account the volume fraction of the dust particles and we have assumed that the initial fluid velocity to be some average velocity. Analytical expressions have been obtained for the velocity components of both of the dust and the fluid. The influence of mass concentration and relaxation time of the dust on the velocity have been clearly pointed out. This problem finds its application in the blood flow through Arteries.

2. OBJECTIVES:

1. To know the behavior of blood flow

2. To know the effect of time on velocity of fluid

3. METHODOLOGY:

The Laplace transform method is use in this research paper and result discuss through graphs using Origion software.

4. BASIC EQUATIONS OF MOTION AND THEIR SOLUTION:

Consider the flow of in incompressible dusty fluid through a circular pipe of radius a. Let the flow be induced by an impulsive pressure gradient along the axis of the pipe taken as x – axis. The equations of motion of unsteady viscous fluid with uniform distribution of dust particles are given by (Saffman, 1962)

$$\rho\left(1-\phi\right)\frac{\partial u^*}{\partial t^*} = -\frac{\partial p^*}{\partial x^*} + \mu\left(\frac{\partial^2 u^*}{\partial r^{*2}} + \frac{1}{r^*}\frac{\partial u^*}{\partial r^*}\right) + NK\left(v^* - u^*\right) \quad (1)$$

$$Nm\frac{\partial v^*}{\partial t^*} = NK\left(u^* - v^*\right) \quad (2)$$

Where u^*, v^* are the components of the fluid and particle velocity in the direction of x- axis which is taken along the length of the tube. The last term on the right hand side of equation (1) represents the force exerted by the particles on the flow while the term on the right hand side of equation (2) is a similar force term exerted by the fluid on the particles. $f = \frac{Nm}{\rho}$ is the ratio of the mass density of the particles and the fluid, commonly known as mass concentration of the dust particles. $\beta = \frac{a}{U\tau}, \tau = \frac{m}{k}$ is the relaxation time of the particles. ρ is the fluid density, μ is the viscosity of fluid, N is constant number

density of the particles, ϕ is volume fraction of particle phase, p is the pressure of fluid. K is stokes resistance coefficient. (i.e. K = 3 μ π d, d is particle diameter) and m is mass of each particle, r^* is the radial distance.

Initial and boundary conditions are

$$u^* = U\,,\ v^* = U \ \text{ at } \ t^* \le 0\,,\ 0 \le r^* \le a \tag{3}$$

$$u^* = 0 \ \text{ at } \ t^* > 0, \quad r^* = a \tag{4}$$

Introducing the following dimensionless variables:

$$u = \frac{u^*}{U},\ v = \frac{v^*}{U},\ r = \frac{r^*}{a},\ p = \frac{p^* - p_0^*}{\frac{1}{2}\rho U^2},\ t = \frac{t^* U}{a},\ x = \frac{x^*}{a}$$

Where U being a reference velocity and p_0^* constant hydrostatic pressure, Equations (1) and (2) reduce to

$$(1-\phi)\frac{\partial u}{\partial t} = F(t) + \frac{2}{R}\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r}\right) + f\ \beta\,(v-u)$$

(5)

$$\frac{\partial v}{\partial t} = \beta\,(u-v)$$

(6)

Where $F(t) = \frac{1}{2}\frac{\partial p}{\partial x}$, $R = \frac{2aU\rho}{\mu}$ - Reynolds Number,

$f = \frac{Nm}{\rho}$ The mass concentration of dust particle (ratio of the mass density of the particles and the fluid), $\beta = \frac{a}{U\tau}$, $\left(\tau = \frac{m}{k}\right)$ - particle relaxation time

The pertinent initial and boundary conditions are

$$u = v = 1 \ \text{ at } \ t \le 0\,, 0 \le r < 1 \tag{7}$$

$$u = 0 \ \text{ at } \ t > 0,\ r = 1 \tag{8}$$

Applying Laplace transform with respect to time t and using the initial conditions given in Equation (7), Equations (5) and (6) reduce to

$$(1-\phi)\left(s\,\bar{u}(s) - 1\right) = \bar{F}(s) + \frac{2}{R}\left(\frac{d^2\bar{u}}{dr^2} + \frac{1}{r}\frac{d\bar{u}}{dr}\right) + f\ \beta\,(\bar{v} - \bar{u}) \tag{9}$$

$$s\,\bar{v}(s) - 1 = \beta\,(\bar{u} - \bar{v}) \tag{10}$$

Where $\{\bar{u}(s)\,,\ \bar{v}(s),\ \bar{F}(s)\} = \int_0^\infty \{u(t),\ v(t),\ F(t)\}\, e^{-st}\ dt$

From (10), we get

$$\bar{v}(s) = \frac{\beta}{s+\beta}\bar{u} + \frac{1}{s+\beta} \tag{11}$$

Substituting the value of $\bar{v}$ in (9), we get

$$\frac{d^2\bar{u}}{dr^2} + \frac{1}{r}\frac{d\bar{u}}{dr} - \frac{R}{2}b\,\bar{u} = -\frac{R}{2}\bar{F}_1 \tag{12}$$

Where $b = \frac{f\beta s}{s+\beta} + (1-\phi)s$, $\bar{F}_1 = \vec{F} + (1-\phi) + \frac{f\beta}{s+\beta}$

Solving Eq. (12) satisfying boundary condition $\bar{u} = 0$ at r = 1 and the regularity condition at r = 0 we get

$$\bar{u} = \frac{\bar{F}_1}{b} - \frac{\bar{F}_1}{b}\frac{J_0\left(ir\sqrt{\frac{Rb}{2}}\right)}{J_0\left(i\sqrt{\frac{Rb}{2}}\right)} \tag{13}$$

Using the convolution theorem and Cauchy's residue theorem, inverse transform of Eq. (13) can be written as

$$u(t) = \sum_{n=1}^{\infty} A_n \int_0^t \exp(s_n\,\tau)\ F(t-\tau)\ d\,\tau + \frac{A_n b_n}{s_n}\exp(s_n\,\tau) \tag{14}$$

Where $A_n = \frac{2}{\lambda_n y_n}\frac{J_0(\lambda_n r)}{J_1(\lambda_n)}$ (15)

λ_n 's are the zeros of the Bessel function $J_0\left(i\sqrt{\frac{Rb}{2}}\right)$ i.e $b_n R = -2\lambda_n^{\ 2}$, hence $b_n(s_n+\beta) = f\beta\ s_n + s_n(1-\phi)\,(s_n+\beta)$ and $y_n = (1-\phi) + \frac{f\beta^2}{(s_n+\beta)^2}$.

Applying inverse Laplace transform to Eq. (11), we get

$$v(t) = \exp(-\beta\ t) + \beta\int_0^t u(\tau)\ \exp\left(-\beta(t-\tau)\right)d\tau \tag{16}$$

5. IMPULSIVE PRESSURE GRADIENT CASE:

Consider a particular case of the impulsive pressure gradient in the form F(t) = P H (t), where P represents the constant adverse pressure gradient and H (t) is Heaviside function of time t.

For this case, the fluid velocity given in Eq. (14) can be written as

$$u(t) = \sum_{n=1}^{\infty}\frac{A_n P}{s_n}\left(\exp(s_n\ t) - 1\right) + \frac{A_n b_n}{s_n}\exp(s_n\ t)$$

(17)

Substituting (17) in (16), we get

$$v(t) = \exp(-\beta\ t) + \sum_{n=1}^{\infty}\frac{A_n P\beta}{s_n}\left(\frac{\exp(s_n t) - \exp(-\beta t)}{s_n+\beta} - \frac{(1-\exp(-\beta t))}{\beta}\right) + \frac{\beta A_n b_n}{s_n}\left(\frac{\exp(s_n t) - \exp(-\beta t)}{s_n+\beta}\right)$$

(18)

6. INSTANTANEOUS RATE OF DISCHARGE:

The instantaneous rate of discharge of fluid is given by

$$Q_u = \frac{Q^*}{Q_s} = 2\int_0^1 urdr \qquad (19)$$

Similarly the dimensionless instantaneous rate of discharge for particle is

$$Q_v = 2\int_0^1 vrdr$$

(20)

Using the values of u and v from Equations (17) and (18) in Eqs. (19) and (20) respectively, the time dependent rate of discharge of fluid and particle phase are obtained.

7. RESULT AND DISCUSSION:

In order to discuss results, numerical computation for Equations (17) and (18) has been made on using typical values of the dimensionless parameters e.g. volume fraction ϕ, mass concentration f, adverse pressure gradient P, Reynolds number R, and for various values of time t. In order to compute the series solution for u- fluid velocity and v – particle velocity, we have considered the first 275 zeros of Bessel function of Zeroth order and first kind numerically. Considering the density ration $\frac{\rho_s}{\rho} \approx O(10)$ and the parameter β = 1.0, it is noted that $\phi \cong \frac{f}{10}$. The fluid velocity and rate of discharge at different times have been computed for various values of the parameter f, (i.e. ϕ also) and they are presented in Figures. 1a – 4a.

Figure1a shows the velocity profiles of fluid and particle phase for various values of f. From the graphs we find that the velocity of the fluid as well as that of the particle phase decreases with the increase of mass concentration f. i.e. of particle loading and volume fraction ϕ. Comparison of the graphs shown in Fig.1a revealed that the velocity of the particle phase is always less than that of the fluid phase, but near the wall velocities of particle and the fluid phase approach to be equal to each other as f increases. It is also noted that velocities of both fluid and dust particles are maximum on the axis of the tube and fluid particles moves faster than the dust particles everywhere in the tube.

Figure 2a shows the distribution of velocity of the fluid and that of the particle phase at the center of the pipe with respect to f for different values of t. The difference between the velocities of fluid and particle phase tends to zero as time increases and they are nearly equal at for $t \geq 100$.

Figure 3a gives the graphs of the velocity of fluid and particle phase against time t for various values of particle loading f. From figure it is observed that the velocities of fluid and particle phase first increases with time due to acceleration up to a constant value for large time as f increases and the steady state is reached at time $t \geq 100$ for both fluid and particle phase.

Figure 4a shows the growth of discharge rates of fluid and particle phase with time. It is seen that these rates of discharge decreases first with time and increase with increase of f (or ϕ). For large time i.e. $t \geq 100$ the discharge rates attain the steady state due to the steadiness of velocity for both fluid and particle phase. Further, the time to reach the steady state increases with the increase of f.

Also we compare these results with the results obtained by Datta and Dalal [10] and they are presented in Figures. 1 – 4. We observed the changes in fluid and particle phases and also the discharge rate of fluid and particle phase between $20 \leq t \leq 100$.

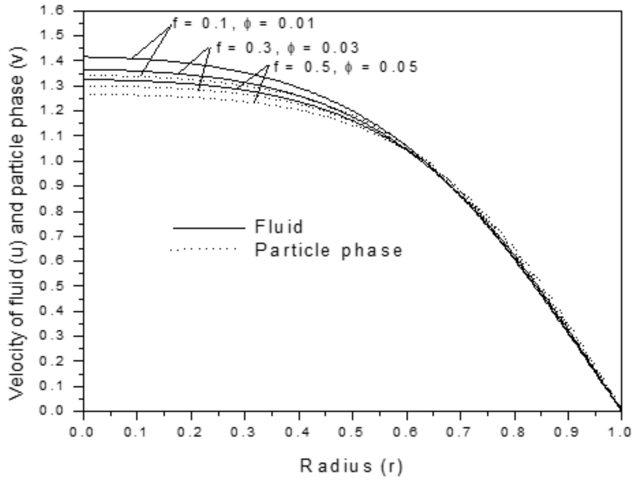
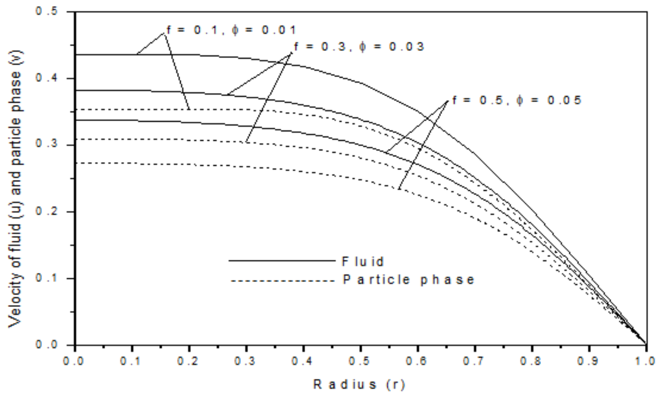


Fig.1a. Velocity distribution of fluid and particle phase when t = 5, p = 0.1 β = 1.0, R = 150



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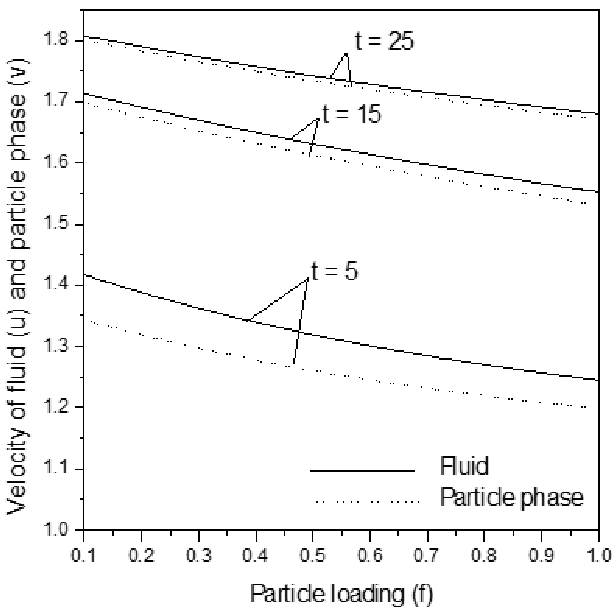

Fig.2a. Variation of central velocity of fluid and particle phase with f for various times when p = 0.1, R = 150, β = 1.0

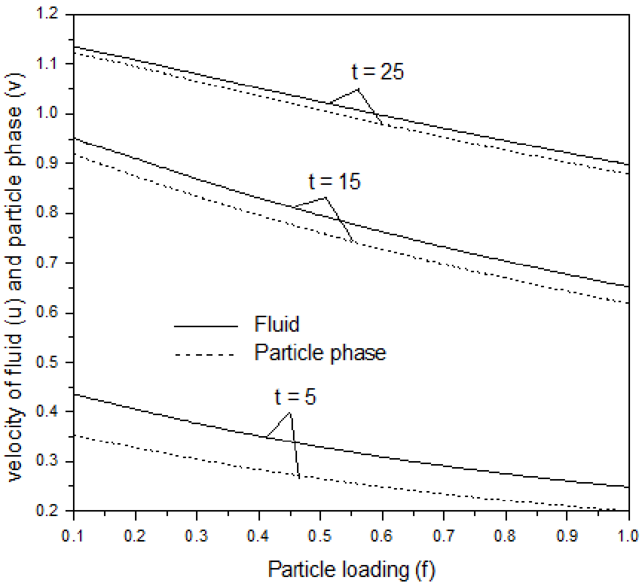

Fig.2. Variation of central velocity of fluid and particle phase with f for various times when p = 0.1, R = 100, β=1.0

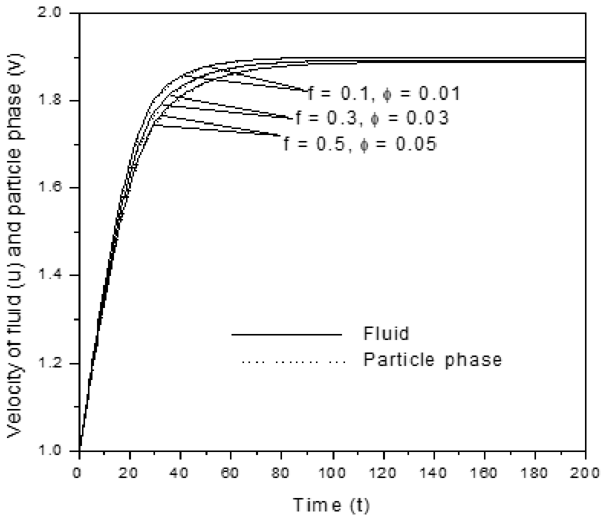

Fig.3a. Time dependent central velocity of fluid and particle phase for various t when p = 0.1, R = 150, β = 1.0

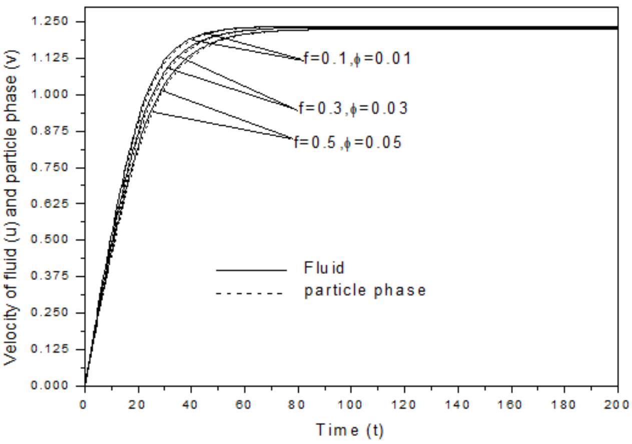

Fig.3. Time dependent central velocity of fluid and particle phase for various f when p = 0.1, R = 100 β=1.0

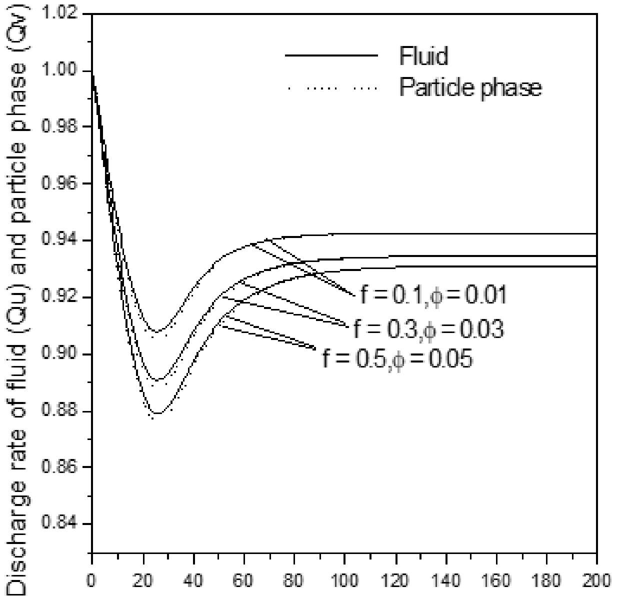

Fig.4a. Variation of discharge rate of flluid and particle phase with time when p = 0.1, R = 150, β = 1.0

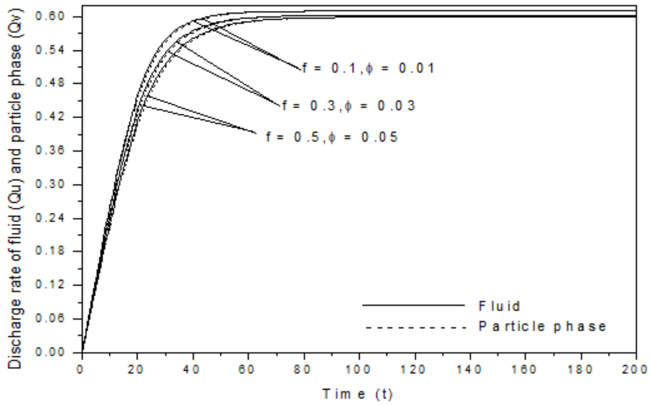

Fig.4. Variation of discharge rate of fluid and particle phase with time when P = 0.1,R = 100,β= 1.0

REFERENCES:

1. Uchida, S. (1956). The pulsating viscous flow superposed on the steady laminar motion of incompressible fluid in a circular pipe Math, Physics. 7, 403-422
2. Saffman, P.G. (1962). On the stability of laminar flow of a dusty gas. J. Fluid Mech.13, 120-128
3. Nayfeh, A.H. (1966). Oscillating two-phase flow through a rigid pipe. A.I.A.A.J. 4, 1868- 1870.
4. Sambasiva Rao, P. (1969). Unsteady flow of a dusty viscous liquid through circular cylinder, Def.Sci.j, 19,135-138,
5. Dube, S.N. and Srivastave, L.P. (1972). Unsteady flow of dusty viscous liquid in a channel and a pipe, Defence Science Journal, 22, 195-198
6. Reddy, Y. B. (1972). Flow of a Dusty liquid through rectangular channel, Defence Science Journal, ,22, 149-152
7. Walburn, F.J. and Schneck, D.J. (1976). A Constitutive equation for whole human blood. Biorheology,13, 201-210
8. Kishore, N., Pandey, R.D. (1977). On the flow of a dusty viscous liquid through a circular pipe. Proc. Indian Acad. Science, 85A, 299-302
9. Nag, S. K., Jana, R.N. (1981). Oscillating two – phase flow in an elastic tube. Acta. Mech. 41, 121-128
10. Ramana Prasad, V.V. and Pattabhi Ramacharyulu, N.Ch. (1981). Couette flow of a Dusty Incompressible Fluid with one of the Horizontal moving boundaries suddenly stopped., Defence Science Journal, 31(3), 231-240
11. Reddy, J.N. and Padhye, V.A. (1988). A Penalty finite element model for axisymmetric flows of non-Newtonian fluids. Num. Math. Part. Diff. Eq. 4, 33-56.
12. Mann, D.E. and Tarbell, J.M. (1990). Flow of non-Newtonian blood analog fluids in rigid curved and straight artery models. Biorheology, 27,711-733.

13. Ghosh, A.K., Khan, A.R., and Debnath, L (1990). On a pulsatile flow of a two-phase viscous fluid in a tube of elliptic cross section. International Journal Mathematics and Mathematic Science, 13(4), 669-676.
14. Datta, N. and Dalal, D.C. (1992). Unsteady flow of dusty fluid through a circular pipe with impulsive pressure gradient. Acta Mechanica, 95, 51-57 (1992).
15. Andersson, H.I. and Tiseth, K.L. (1992). Start-up flow in a pipe following the sudden imposition of a constant flow rate. Chem. Engg. Comm, 112, 121-133.
16. Hazeem Ali Attia (2006). Hall effect on the flow of a Dusty Bingham fluid in a circular pipe, Turkish Journal of English Environment. Science, 30, 14-21.
17. Gireesha, C.S. Bagewadi and B.C. Prasanna kumar (2007). Flow of unsteady dusty fluid under varying pulsatile pressure gradient in anholonomic co-ordinate system, EJTP, 4, 14, 9-16.
18. Rashmi, S., V.Kavitha, B.Saba Roohi, Gurumurthy, B.J.Gireesha and C.S. Bagewadi (2007). Unsteady flow of a dusty fluid between two oscillating plates under varying constant pressure gradient, Novi Sad Journal of Mathematics, 37(2), 25- 34.
19. Siddapasappa, Y.Venkateshappa, B.Rudraswamy, B.J. Gireesha and K.R. Gopinath. (2008). Viscous dusty fluid flow with constant velocity magnitude, EJTP, 5, 237- 252.